Question 2 **(6 marks)**

For a set of data values that are normally distributed, approximately 68% of the values will lie within one standard deviation of the mean, approximately 95% of the values will lie within two standard deviations of the mean and approximately 99.7% of the values will lie within three standard deviations of the mean.

If the heights of a large group of women are normally distributed with a mean μ = 163 cm and standard deviation σ = 7 cm, use the above information to answer the following questions:

(a) A statistician says that almost all of the women have heights in the range 142 cm to 184 cm. Comment on her statement. (2 marks)

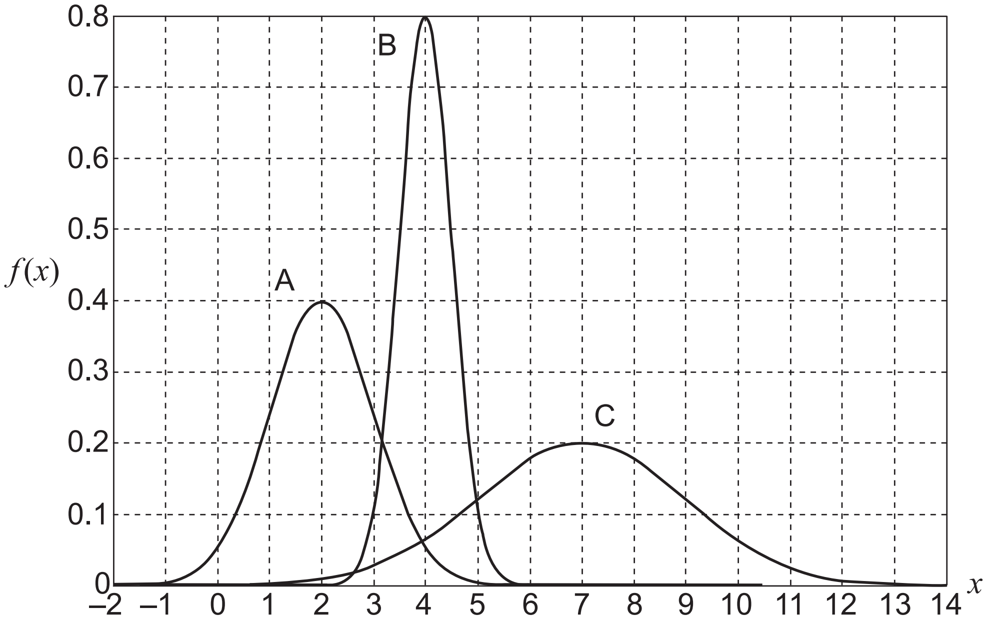
(b) Approximately what percentage of women in the group has a height greater than 170 cm? (2 marks)

(c) Approximately 2.5% of the women are shorter than what height? (2 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 6 **(7 marks)**

(a) The graphs of three normal distributions are displayed below. The distributions have been labelled A, B and C.

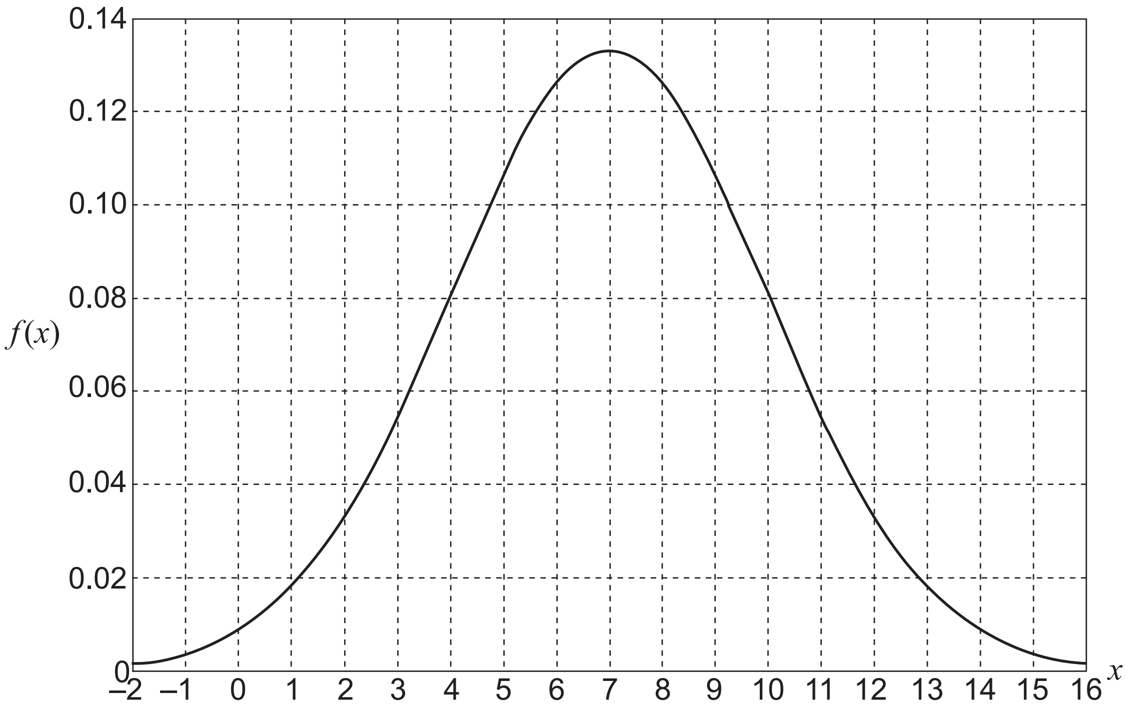


(i) What is the mean of distribution A? (1 mark)

(ii) Which of the distributions has the largest standard deviation? Justify your answer. (1 mark)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

(b) A random variable X is normally distributed. The distribution of X is graphed below.



(i) Shade the region with area corresponding to $\mathrm{P}(6 \le X \le 9)$. (1 mark)

(ii) Is $\mathrm{P}(6 \le X \le 9) \ge 0.5$? Justify your answer. (2 marks)

(c) A random variable Y has probability $\mathrm{P}(Y \ge 2) > \mathrm{P}(Y > 2)$. Explain whether it is possible for the distribution of Y to be normal or binomial. (2 marks)

Section Two: Calculator-assumed **65% (97 Marks)**

This section has **10** questions. Answer **all** questions. Write your answers in the spaces provided.

Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 100 minutes.

Question 8 **(7 marks)**

The weight, X, of chicken eggs from a farm is normally distributed with mean 60 g and standard deviation 5 g. Eggs with a weight of more than 67 g are classed as 'jumbo'.

(a) What proportion of eggs from the farm are 'jumbo'? (2 marks)

(b) What proportion of 'jumbo' eggs are less than 75 g in weight? (3 marks)

(c) The heaviest 0.05% of eggs fetch a higher price. What is the minimum weight of these eggs? (2 marks)

Section Two: Calculator-assumed **65% (100 Marks)**

This section has **10** questions. Answer **all** questions. Write your answers in the spaces provided.

Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 100 minutes.

Question 8 **(9 marks)**

The weights W (in grams) of carrots sold at a supermarket have been found to be normally distributed with a mean of 142.8 g and a standard deviation of 30.6 g.

(a) Determine the percentage of carrots sold at the supermarket that weigh more than 155 g. (2 marks)

Carrots sold at the supermarket are classified by weight, as shown in the table below.

Classification	Small	Medium	Large	Extra large
Weight W (grams)	$W \leq 110$	$110 < W \leq 155$	$155 < W \leq 210$	$W > 210$
$\mathrm{P}(W)$		0.5131	0.3310	

(b) Complete the table above, providing the missing probabilities. (2 marks)

(c) Of the carrots being sold at the supermarket that are **not** of medium weight, what proportion is small? (2 marks)

The supermarket sells bags of mixed-weight carrots, with 12 randomly-selected carrots placed in each bag.

(d) If a customer purchases a bag of mixed-weight carrots, determine the probability that there will be at most two small carrots in the bag. (3 marks)

Question 11 **(8 marks)**

A pizza company runs a marketing campaign based on the delivery times of its pizzas. The company claims that it will deliver a pizza in a radius of 5 km within 30 minutes of ordering or it is free. The manager estimates that the actual time, T, from order to delivery is normally distributed with mean 25 minutes and standard deviation 2 minutes.

(a) What it the probability that a pizza is delivered free? (1 mark)

(b) On a busy Saturday evening, a total of 50 pizzas are ordered. What is the probability that more than three are delivered free? (2 marks)

The company wants to reduce the proportion of pizzas that are delivered free to 0.1%.

(c) The manager suggests this can be achieved by increasing the advertised delivery time. What should the advertised delivery time be? (2 marks)

After some additional training the company was able to maintain the advertised delivery time as 30 minutes but reduce the proportion of pizzas delivered free to 0.1%.

(d) Assuming that the original mean of 25 minutes is maintained, what is the new standard deviation of delivery times? (3 marks)

Question 12 **(19 marks)**

The manager of the mail distribution centre in an organisation estimates that the weight, x (kg), of parcels that are posted is normally distributed, with mean 3 kg and standard deviation 1 kg.

(a) What percentage of parcels weigh more than 3.7 kg? (2 marks)

(b) Twenty parcels are received for posting. What is the probability that at least half of them weigh more than 3.7 kg? (3 marks)

The cost of postage, (\$) y, depends on the weight of a parcel as follows:

- a cost of \$5 for parcels 1 kg or less
- an additional variable cost of \$1.50 for every kilogram or part thereof above 1 kg to a maximum of 4 kg
- a cost of \$12 for parcels above 4 kg.

(c) Complete the probability distribution table for Y. (4 marks)

x	≤ 1	$1 < x \leq 2$	$2 < x \leq 3$	$3 < x \leq 4$	$x > 4$
y	\$5				
$P(Y = y)$					

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(d) Calculate the mean cost of postage per parcel. (2 marks)

(e) Calculate the standard deviation of the cost of postage per parcel. (3 marks)

(f) If the cost of postage is increased by 20% and a surcharge of $1 is added for all parcels, what will be the mean and standard deviation of the new cost? (3 marks)

(g) Show one reason why the given normal distribution is not a good model for the weight of the parcels. (2 marks)

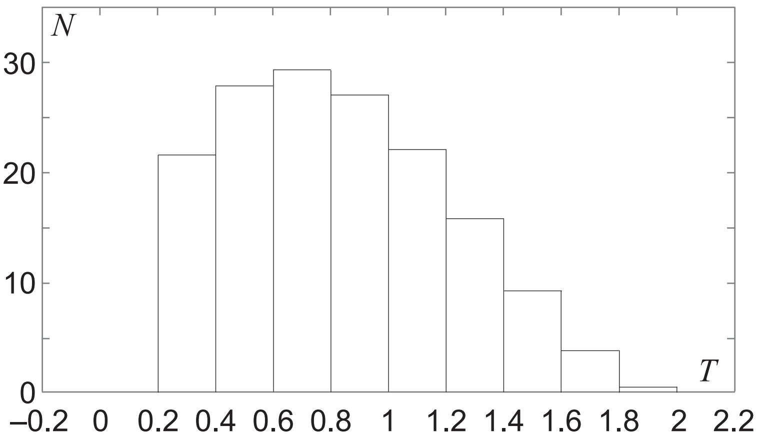
Question 16 **(7 marks)**

A large refrigerator in a scientific laboratory is always required to maintain a temperature between $0\ °C$ and $1\ °C$ to preserve the integrity of biological samples stored inside. A scientist working in the laboratory suspects that the refrigerator is not maintaining the required temperature and decides to record the temperature every hour for seven days. Based on these measurements, the scientist concludes that the temperature, T, in the refrigerator is normally distributed with a mean of $0.8\ °C$ and a standard deviation of $0.4\ °C$.

(a) Temperature in degrees Fahrenheit, T_f, is given by $T_f = \frac{9}{5}T + 32$. Determine the mean and standard deviation of the refrigerator temperature in degrees Fahrenheit. (2 marks)

(b) Determine the probability that the refrigerator temperature is above $1\ °C$. Give your answer rounded to four decimal places. (1 mark)

The histogram of data gathered by the scientist is shown below. N denotes the number of observations in each temperature interval.



(c) Do you agree that the normal distribution was an appropriate model to use? Provide a reason to justify your response. (2 marks)

An alternative probability density function proposed to model the refrigerator temperature, in degrees Celcius, is given by:

$$p(t) = \frac{3}{4}t^3 - 3t^2 + 3t\,,\ \ 0 \leq t \leq 2$$

(d) Determine the probability that the refrigerator temperature is above $1\ °\mathrm{C}$ using the new model. (2 marks)

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Question 18 **(6 marks)**

The waiting times at a Perth Airport departure lounge have been found to be normally distributed. It is observed that passengers wait for less than 55 minutes, 5% of the time, while there is a 13% chance that the waiting times will be greater than 100 minutes.

(a) Determine the mean and standard deviation for the waiting times at Perth Airport departure lounge. (5 marks)

(b) Determine the probability that the waiting time will be between 75 and 90 minutes. (1 mark)

Question 19 **(12 marks)**

A global financial institution transfers a large aggregate data file every evening from offices around the world to its Hong Kong head office. Once the file is received it must be processed in the company's data warehouse. The time T required to process a file is normally distributed with a mean of 90 minutes and a standard deviation of 15 minutes.

(a) An evening is selected at random. What is the probability that it takes more than two hours to process the file? (2 marks)

(b) What is the probability that the process takes more than two hours on two out of five days in a week? (3 marks)

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The company is considering outsourcing the processing of the files.

(c) (i) A quotation for this job from an IT company is given in the table below. Complete the table. (1 mark)

Job duration (minutes)	$T \le 60$	$60 < T < 120$	$T \ge 120$
Probability			
Cost Y ($)	200	600	1200

(ii) What is the mean cost? (2 marks)

(iii) Calculate the standard deviation of the cost. (2 marks)

(iv) In the following year, the cost (currently $\$Y$) will increase due to inflation and also the introduction of an additional fixed cost, so the new cost $\$N$ is given by: $N = aY + b$. In terms of a and/or b, state the mean cost in the following year and the standard deviation of the cost in the following year. (2 marks)

Question 2 **(6 marks)**

For a set of data values that are normally distributed, approximately 68% of the values will lie within one standard deviation of the mean, approximately 95% of the values will lie within two standard deviations of the mean and approximately 99.7% of the values will lie within three standard deviations of the mean.

If the heights of a large group of women are normally distributed with a mean μ = 163 cm and standard deviation σ = 7 cm, use the above information to answer the following questions:

(a) A statistician says that almost all of the women have heights in the range 142 cm to 184 cm. Comment on her statement. (2 marks)

Solution
Her comment is appropriate as the range corresponds to 3 standard deviations above and below the mean, which equates to approximately 99.7% of the group.
Specific behaviours
✓ states that the comment is appropriate ✓ refers to the standard deviation and 99.7%

(b) Approximately what percentage of women in the group has a height greater than 170 cm? (2 marks)

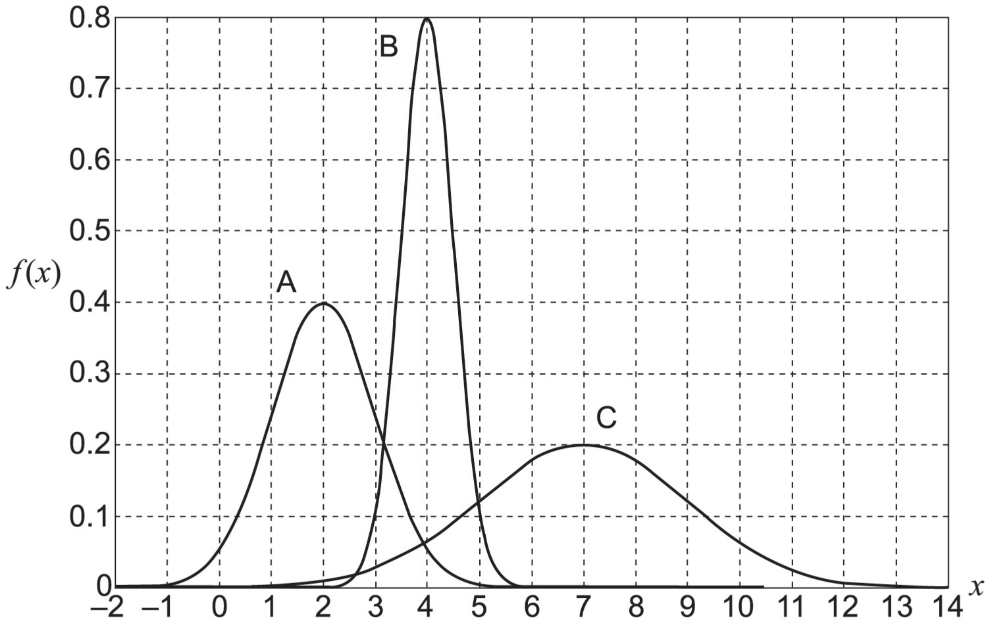
Solution
$170-163=7 \Rightarrow 1 \text{ SD above}$ $\text{Percentage} = \dfrac{100-68}{2}$ $=16$
Specific behaviours
✓ states 1 standard deviation above ✓ determines correct percentage

(c) Approximately 2.5% of the women are shorter than what height? (2 marks)

Solution
$\text{percentage} = 100-2\times 2.5$ $=95\%$ $2 \text{ SDs below} = 163-14$ $=149 \text{ cm}$
Specific behaviours
✓ determines 95% ✓ states height

Question 6 **(7 marks)**

(a) The graphs of three normal distributions are displayed below. The distributions have been labelled A, B and C.



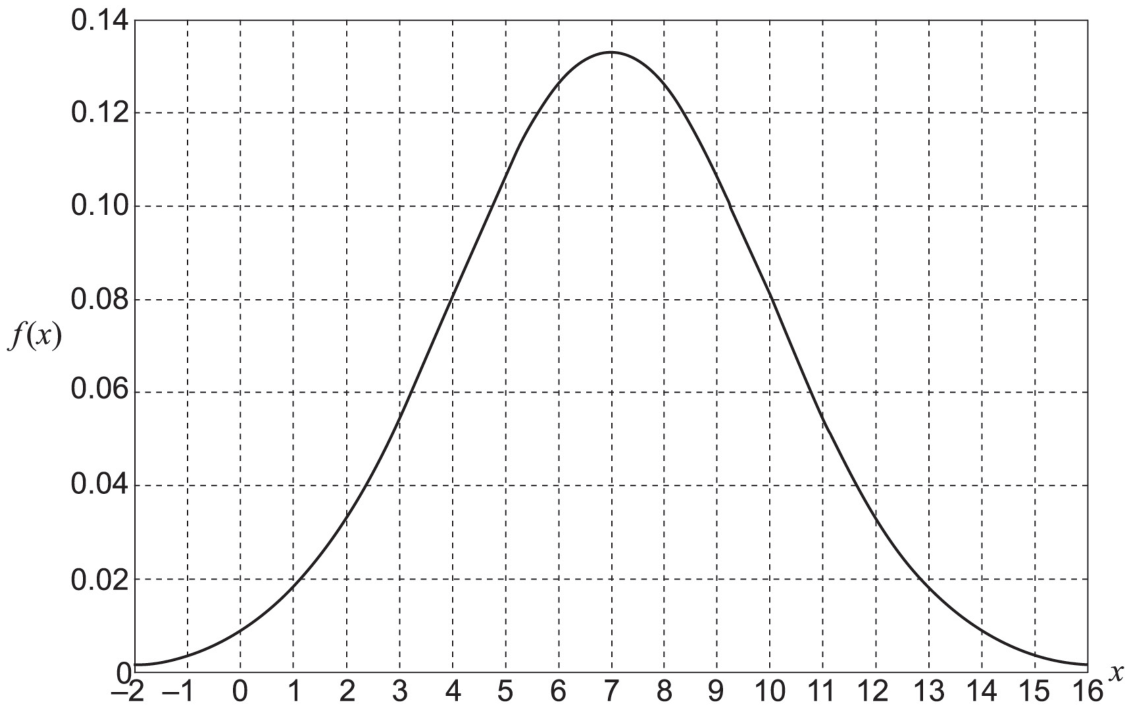
(i) What is the mean of distribution A? (1 mark)

Solution
Mean = 2
Specific behaviours
✓ determines mean of A

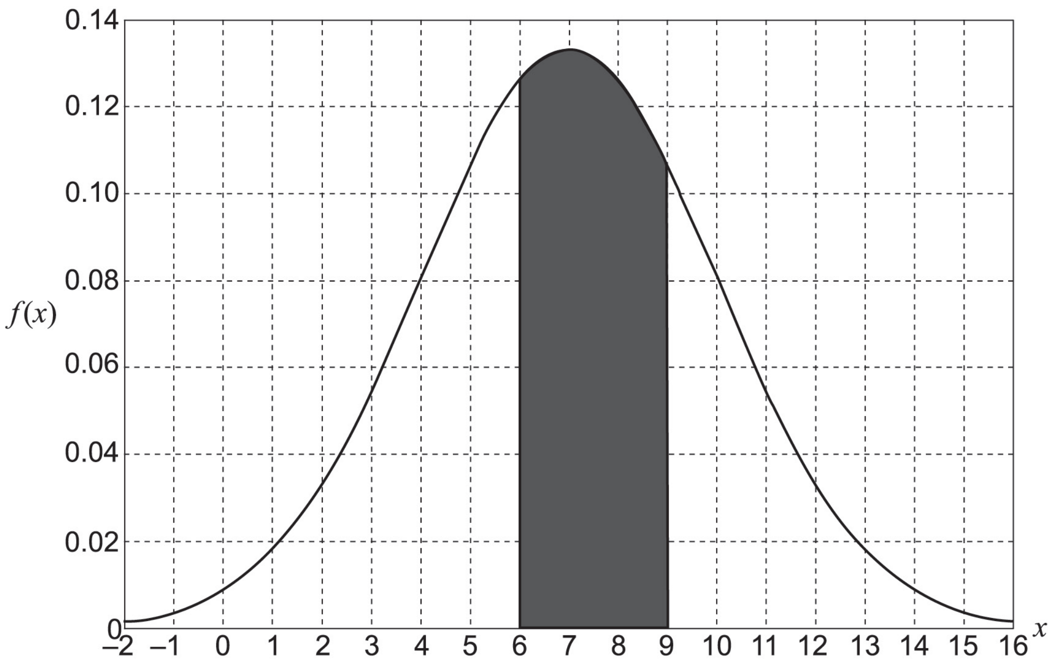
(ii) Which of the distributions has the largest standard deviation? Justify your answer. (1 mark)

Solution
C has the largest standard deviation as it is the widest distribution.
Specific behaviours
✓ states that C has the largest standard deviation and provides correct justification

(b) A random variable X is normally distributed. The distribution of X is graphed below.



(i) Shade the region with area corresponding to $\mathrm{P}(6 \le X \le 9)$. (1 mark)

Solution

Specific behaviours
✓ shades correct region

Question 6 (continued)

(ii) Is $\mathrm{P}(6 \le X \le 9) \ge 0.5$? Justify your answer. (2 marks)

Solution
No. The total area below the probability density function is 1, and the region shaded above is less than half of that area (i.e. area is less than 0.5). Hence, it corresponds to a probability that is less than 0.5.
Specific behaviours
✓ states that the probability is not greater than or equal to 0.5 ✓ provides correct justification

(c) A random variable Y has probability $\mathrm{P}(Y \ge 2) > \mathrm{P}(Y > 2)$. Explain whether it is possible for the distribution of Y to be normal or binomial. (2 marks)

Solution
Not normal: a continuous random variable has $\mathrm{P}(Y \ge 2) = \mathrm{P}(Y > 2)$. Since a normally distributed random variable is continuous it follows that Y is not a normally distributed random variable. Could be binomial: $\mathrm{P}(Y \ge 2) > \mathrm{P}(Y > 2)$ for a discrete random variable. Since the binomial distribution is discrete it follows that Y could be a binomially distributed random variable.
Specific behaviours
✓ states that Y could not be normal and provides a correct explanation ✓ states that Y could be binomial and provides a correct explanation

Section Two: Calculator-assumed **65% (97 Marks)**

Question 8 **(7 marks)**

The weight, X, of chicken eggs from a farm is normally distributed with mean 60 g and standard deviation 5 g. Eggs with a weight of more than 67 g are classed as 'jumbo'.

(a) What proportion of eggs from the farm are 'jumbo'? (2 marks)

Solution
$P(X > 67) = 0.08076$
Specific behaviours
✓ states the correct expression for the probability ✓ calculates the probability

(b) What proportion of 'jumbo' eggs are less than 75 g in weight? (3 marks)

Solution
$P(X < 75 \mid X > 67) = \dfrac{P(67 < X < 75)}{P(X > 67)} = \dfrac{0.0794}{0.0808} = 0.9832$
Specific behaviours
✓ writes a conditional probability statement ✓ recognises the restricted domain for 'jumbo' eggs ✓ calculates the probability

(c) The heaviest 0.05% of eggs fetch a higher price. What is the minimum weight of these eggs? (2 marks)

Solution
$P(X > m) = 0.0005$ $m = 76.45$ g
Specific behaviours
✓ writes the correct expression $P(X > m) = 0.0005$ ✓ calculates the minimum weight

Section Two: Calculator-assumed **65% (100 Marks)**

Question 8 **(9 marks)**

The weights W (in grams) of carrots sold at a supermarket have been found to be normally distributed with a mean of 142.8 g and a standard deviation of 30.6 g.

(a) Determine the percentage of carrots sold at the supermarket that weigh more than 155 g. (2 marks)

Solution
$P(W > 155) = 0.3451$ $0.3451 \times 100 = 34.51\%$
Specific behaviours
✓obtains correct probability ✓obtains correct percentage

Carrots sold at the supermarket are classified by weight, as shown in the table below.

Classification	**Small**	**Medium**	**Large**	**Extra large**
Weight W (grams)	$W \le 110$	$110 < W \le 155$	$155 < W \le 210$	$W > 210$
$P(W)$	**0.1418**	0.5131	0.3310	**0.3451 – 0.3310 = 0.0141**

(b) Complete the table above, providing the missing probabilities. (2 marks)

Solution
See table above
Specific behaviours
✓determines one correct probability ✓determines second correct probability

(c) Of the carrots being sold at the supermarket that are **not** of medium weight, what proportion is small? (2 marks)

Solution
$P(\text{Small} \mid \text{not Medium}) = \dfrac{P(\text{Small})}{P(\text{not Medium})} = \dfrac{0.1418}{0.4869} = 0.2912$
Specific behaviours
✓determines correct denominator ✓determines correct numerator and obtains final answer

The supermarket sells bags of mixed-weight carrots, with 12 randomly-selected carrots placed in each bag.

(d) If a customer purchases a bag of mixed-weight carrots, determine the probability that there will be at most two small carrots in the bag. (3 marks)

Solution
Let the random variable Y denote the number of small carrots in a bag. Then $Y \sim \text{Bin}\,(12, 0.1418)$ We need $\text{P}(Y \leq 2) = 0.7637$
Specific behaviours
✓ defines appropriate random variable and states the correct binomial distribution ✓ states the correct probability statement ✓ computes the probability

Question 11 **(8 marks)**

A pizza company runs a marketing campaign based on the delivery times of its pizzas. The company claims that it will deliver a pizza in a radius of 5 km within 30 minutes of ordering or it is free. The manager estimates that the actual time, T, from order to delivery is normally distributed with mean 25 minutes and standard deviation 2 minutes.

(a) What it the probability that a pizza is delivered free? (1 mark)

Solution
$P(T > 30) = P\left(Z > \frac{30-25}{2}\right) = P(Z > 2.5) = 1 - 0.9938 = 0.0062$
Specific behaviours
✓ gives the correct value of the probability

(b) On a busy Saturday evening, a total of 50 pizzas are ordered. What is the probability that more than three are delivered free? (2 marks)

Solution
Let Xdenote the number of pizzas out of 50 that are delivered free. Then $X \sim Bin(50{,}0.0062)$ $P(X > 3) = 1 - P(X \leq 3) = 1 - 0.9997 = 0.0003$
Specific behaviours
✓ states the distribution of the number of pizzas delivered free ✓ computes the probability correctly

The company wants to reduce the proportion of pizzas that are delivered free to 0.1%.

(c) The manager suggests this can be achieved by increasing the advertised delivery time. What should the advertised delivery time be? (2 marks)

Solution
$P(Z > z) = 0.001 \Rightarrow z = 3.0902 \Rightarrow t = 25 + 3.0902 \times 2 = 31.2$ minutes
Specific behaviours
✓ uses a tail probability of 0.001 ✓ calculates the correct value of the delivery time

After some additional training the company was able to maintain the advertised delivery time as 30 minutes but reduce the proportion of pizzas delivered free to 0.1%.

(d) Assuming that the original mean of 25 minutes is maintained, what is the new standard deviation of delivery times? (3 marks)

Solution
$z = \frac{30-25}{\sigma} = 3.0902 \Rightarrow \sigma = \frac{30-25}{3.0902} = 1.6$ minutes
Specific behaviours
✓ uses the correct critical value of the normal distribution ✓ forms the correct equation for σ ✓ solves for σ

Question 12 **(19 marks)**

The manager of the mail distribution centre in an organisation estimates that the weight, x (kg), of parcels that are posted is normally distributed, with mean 3 kg and standard deviation 1 kg.

(a) What percentage of parcels weigh more than 3.7 kg? (2 marks)

Solution
$X \sim N(3, 1)$ $P(X > 3.7) = 0.24196$ 24.2% are greater than 3.7 kg.
Specific behaviours
✓ states weight required greater than 3.7 kg ✓ obtains the correct percentage

(b) Twenty parcels are received for posting. What is the probability that at least half of them weigh more than 3.7 kg? (3 marks)

Solution
Let the random variable M denote the number of parcels that weigh more than 3.7 kg. Then $M \sim Bin(20, 0.24196)$. $P(M \geq 10) = 0.01095$
Specific behaviours
✓ states the distribution as binomial ✓ determines the correct parameters of the distribution ✓ obtains the correct probability

The cost of postage, (\$) y, depends on the weight of a parcel as follows:

- a cost of \$5 for parcels below 1 kg
- a variable cost of \$1.50 for every kilogram or part thereof above 1 kg to a maximum of 4 kg
- a cost of \$12 for parcels above 4 kg.

(c) Complete the probability distribution table for Y. (4 marks)

x	≤ 1	$1 < x \leq 2$	$2 < x \leq 3$	$3 < x \leq 4$	$x > 4$
y	\$5	**\$6.50**	**\$8**	**\$9.50**	**\$12**
$P(Y = y)$	**0.02275 (accept 0.02140)**	**0.13591**	**0.34134**	**0.34134**	**0.15866**

Solution
See table
Specific behaviours
✓ obtains two correct values of y ✓ obtains the other two correct values of y ✓ obtains two correct probabilities ✓ obtains the remaining correct probabilities

Question 12 (continued)

(d) Calculate the mean cost of postage per parcel. (2 marks)

Solution
$E(Y) = 5\times 0.02275 + 6.5\times 0.13591 + 8\times 0.34134 + 9.50\times 0.34134 + 12\times 0.15866$ $= 8.874535$ That is, \$8.87 is the mean cost of postage per parcel.
Specific behaviours
✓ obtains the correct expression for the mean   ✓ obtains the correct value of the mean

(e) Calculate the standard deviation of the cost of postage per parcel. (3 marks)

Solution
$\sigma^2 = (5-8.87)^2\times 0.02275 + (6.5-8.87)^2\times 0.13591 + (8-8.87)^2\times 0.34134$ $+\ (9.5-8.87)^2\times 0.34134 + (12-8.87)^2\times 0.15866$ $= 3.052310889$ $\therefore \sigma = 1.7470864$
Specific behaviours
✓ substitutes into variance formula correctly   ✓ calculates the variance correctly   ✓ calculates the standard deviation correctly

(f) If the cost of postage is increased by 20% and a surcharge of \$1 is added for all parcels, what will be the mean and standard deviation of the new cost? (3 marks)

Solution
The mean will increase by 20% to $1.2\times 8.874535 + 1 = 11.64944$.   The standard deviation increases by 20% to $1.2\times 1.747086 = 2.096504$.
Specific behaviours
✓ states new values will need to be multiplied by 1.2   ✓ correctly determines mean   ✓ correctly determines standard deviation

(g) Show one reason why the given normal distribution is not a good model for the weight of the parcels? (2 marks)

Solution
$P(Y<0) = 0.001349898$ There is a non-zero (small) probability that the weight can be negative, which is not possible.
Specific behaviours
✓ calculates the probability of a weight below 0   ✓ explains that negative weights are not possible here

Question 16 **(7 marks)**

A large refrigerator in a scientific laboratory is always required to maintain a temperature between 0 °C and 1 °C to preserve the integrity of biological samples stored inside. A scientist working in the laboratory suspects that the refrigerator is not maintaining the required temperature and decides to record the temperature every hour for seven days. Based on these measurements, the scientist concludes that the temperature, T, in the refrigerator is normally distributed with a mean of 0.8 °C and a standard deviation of 0.4 °C.

(a) Temperature in degrees Fahrenheit, T_f, is given by $T_f = \frac{9}{5}T + 32$. Determine the mean and standard deviation of the refrigerator temperature in degrees Fahrenheit. (2 marks)

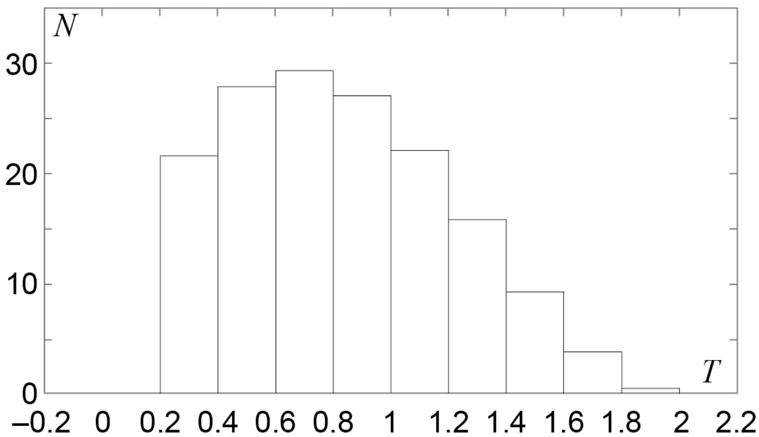
Solution
The mean of T_f is $\mu_{T_f} = \frac{9}{5}\mu_T + 32$ $= \frac{9}{5}\left(\frac{4}{5}\right) + 32$ $= \frac{836}{25} = 33\frac{11}{25} = 33.44$ The standard deviation of T_f is $\sigma_{T_f} = \frac{9}{5}\sigma_T$ $= \frac{9}{5}\left(\frac{2}{5}\right)$ $= \frac{18}{25} = 0.72$
Specific behaviours
✓ determines correct mean   ✓ determines correct standard deviation

(b) Determine the probability that the refrigerator temperature is above 1 °C. Give your answer rounded to four decimal places. (1 mark)

Solution
$P(T > 1) = 0.3085$
Specific behaviours
✓ determines correct probability

Question 16 (continued)

The histogram of data gathered by the scientist is shown below. N denotes the number of observations in each temperature interval.



(c) Do you agree that the normal distribution was an appropriate model to use? Provide a reason to justify your response. (2 marks)

Solution
No. The distribution appears to be skewed to the right (non-symmetric)
Specific behaviours
✓ recognises that the normal distribution was not an appropriate model ✓ justifies conclusion based on lack of symmetry of histogram

An alternative probability density function proposed to model the refrigerator temperature in degrees Celcius, is given by

$$p(t) = \frac{3}{4}t^3 - 3t^2 + 3t\,,\ \ 0 \le t \le 2$$

(d) Determine the probability that the refrigerator temperature is above 1 °C using the new model. (2 marks)

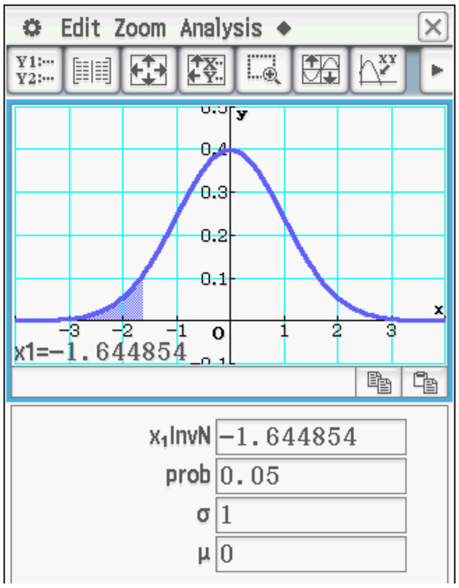
Solution
$P(T \ge 1) = \int_1^2 p(t)\,dt$ $= \int_1^2 \frac{3}{4}t^3 - 3t^2 + 3t\,dt$ $= \left[\frac{3}{16}t^4 - t^3 + \frac{3}{2}t^2\right]_1^2$ $= 1 - \frac{11}{16}$ $= \frac{5}{16}$ {0.3125}
Specific behaviours
✓ identifies integral to determine probability ✓ determines correct probability

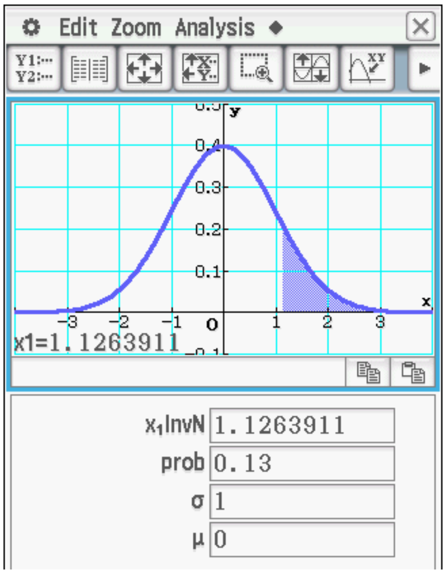
Question 18 **(6 marks)**

The waiting times at a Perth Airport departure lounge have been found to be normally distributed. It is observed that passengers wait for less than 55 minutes, 5% of the time, while there is a 13% chance that the waiting times will be greater than 100 minutes.

(a) Determine the mean and standard deviation for the waiting times at Perth Airport departure lounge. (5 marks)

Solution





$$\left\{\begin{array}{l} -1.64485=\dfrac{55-u}{s} \\ 1.12639=\dfrac{100-u}{s} \end{array}\right|_{s,\,u}$$

$$\{s=16.23821827,\ u=81.70943332\}$$

Specific behaviours
✓ determines both z scores ✓ sets up at least one equation with mean and standard deviation ✓ sets up two equations with mean and standard deviation ✓ solves for mean ✓ solves for standard deviation

(b) Determine the probability that the waiting time will be between 75 and 90 minutes. (1 mark)

Solution
normCDf(75, 90, 16.2382, 81.709) 0.3554354358

Specific behaviours
✓ determines probability

Question 19 **(12 marks)**

A global financial institution transfers a large aggregate data file every evening from offices around the world to its Hong Kong head office. Once the file is received it must be processed in the company's data warehouse. The time T required to process a file is normally distributed with a mean of 90 minutes and a standard deviation of 15 minutes.

(a) An evening is selected at random. What is the probability that it takes more than two hours to process the file? (2 marks)

Solution
$T \sim N(90, 15^2)$ so $P(T > 120) = P\left(Z > \frac{120-90}{15}\right) = P(Z > 2) = 0.0228$
Specific behaviours
✓ writes correct probability statement ✓ calculates correct probability

(b) What is the probability that the process takes more than two hours on two out of five days in a week? (3 marks)

Solution
Let the random variable X denote the number of days out of 5 that the process takes more than 2 hours. Then $X \sim Bin(5,0.0228)$. $P(X = 2) = \binom{5}{2} 0.0228^2(1 - 0.0228)^3 = 0.00485$
Specific behaviours
✓ identifies binomial distribution ✓ uses correct parameters for binomial ✓ calculates correct probability

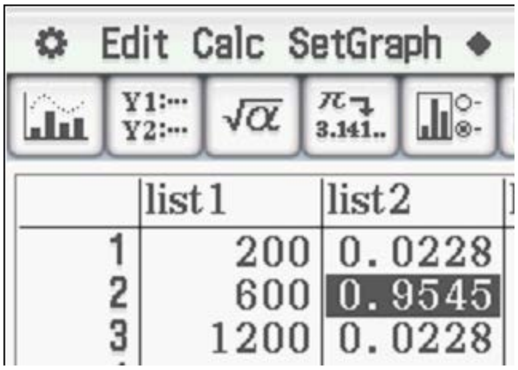
Question 19 (continued)

The company is considering outsourcing the processing of the files.

(c) (i) A quotation for this job from an IT company is given in the table below.
Complete the table. (1 mark)

Solution

Job Duration T (minutes)	$T \leq 60$	$60 < T < 120$	$T \geq 120$
Probability	**0.0228**	**0.9545**	**0.0228**
Cost Y ($)	200	600	1200

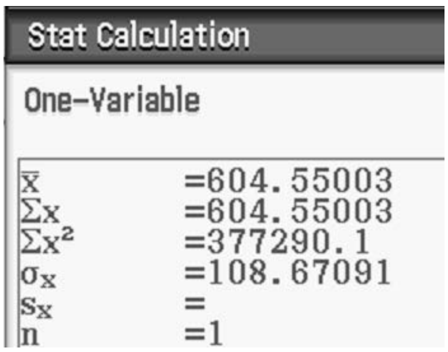


Specific behaviours
✓ calculates the probabilities correctly

(ii) What is the mean cost? (2 marks)

Solution

The probability distribution of Y is given below.



mean cost = $200 \times 0.0228 + 600 \times 0.9545 + 1200 \times 0.0228$
$= \$604.55$

Specific behaviours
✓ writes an expression for the mean cost per file ✓ calculates the mean correctly